

bounds for the orthonormal polynomials, bounds for L_p norms of orthonormal polynomials, and estimates for the spacing of their zeros. The bounds hold uniformly on the whole interval $(-1, 1)$, thus not only asymptotically almost everywhere or locally uniformly on closed subsets of $(-1, 1)$. Clever use is made of logarithmic potential theory, including a chapter on the discretization of a potential. In this respect the reviewer also wants to draw attention to the recent monograph of V. Totik: *Weighted approximation with varying weight* (see the book review in this issue), who also uses logarithmic potential theory and discretization of potentials to construct appropriate approximations which can be used to obtain bounds for orthogonal polynomials.

This is a long research paper which contains important ideas and which will be essential to anyone interested in the analysis of orthogonal polynomials, in particular in the asymptotic theory for orthogonal polynomials on $[-1, 1]$ not satisfying Szegő's condition.

WALTER VAN ASSCHE

F. ALTOMARE AND M. CAMPITI, *Korovkin-type Approximation Theory and Its Applications*, de Gruyter Studies in Mathematics 17, Walter de Gruyter, Berlin, 1994, xi + 627 pp.

If one was to compile a list of *Famous Theorems in Approximation Theory*, then one would have to include P. P. Korovkin's theorem about the convergence of positive linear operators. The statement of the theorem shocks the newcomer to approximation theory, the power of the theorem is obvious, and its proof is elegant.

Since the publication of this result by Korovkin in 1953, there have been many developments which refine or generalize Korovkin's original classical theorem. Numerous review articles and books in the area have been published. The main purpose of this new book by Altomare and Campiti is, in the words of the authors, to present "*a modern and comprehensive exposition of the main aspects of the theory in spaces of continuous functions (vanishing at infinity, respectively) defined on a compact set (a locally compact space, respectively) together with its main applications.*"

The plan of the book is surprising. I expected that the book would begin with Korovkin's original theorem, show a few well-known applications, and then branch out into more general settings. But, instead, it opens with a lengthy summary (73 pp.) of relevant aspects of Radon measures, locally convex spaces, probability theory, and stochastic processes. This introduction prepares us for Chapter 2 which outlines Korovkin-type theorems for bounded, positive Radon measures on a locally compact Hausdorff space. This chapter concludes with a discussion of Choquet boundaries and their relation to Korovkin-type theorems. Chapter 3 continues in the abstract vein established in the preceding chapter. Results in Chapter 3 are centered on the convergence of equicontinuous nets of positive linear operators to some fixed, positive linear operator (not necessarily the identity operator). Chapter 4 has applications in mind because it deals with convergence of nets of positive linear operators to the identity operator. This chapter introduces the notion of Korovkin closure of a set (of test functions) which is central to the applications of the general theory to classical problems in approximation by positive linear operators. This chapter contains Korovkin's famous $\{1, x, x^2\}$ -theorem. Chapter 5 is entitled "Applications to Positive Approximation Processes on Real Intervals." Here we see applications of Korovkin's theorem to the study of approximation by positive linear operators associated with names such as Bernstein, King, Baskakov, Stancu, Cheney-Sharma, Gauss-Weierstrass, Hermite-Fejér (called Fejér-Hermite in this book), Szász-Mirakjan, Mastroianni, and others. A very interesting part (pp. 283-293) of this chapter is devoted to discussing the general relationship between positive approximation processes and probabilistic methods. The final chapter of the book deals with applications of the convergence of sequences of positive linear operators to some aspects of the theory of partial differential equations and to the theory of Markov processes.

I must include some remarks on the book's infrastructure. On pp. 8 and 9, the authors present novel descriptions of the interdependence of sections. We find, on p. 8, a lower triangular matrix where the entry in the position (i, j) indicates the level of dependence of section i on section j . On p. 9, the authors explain the dependencies further with a series of detailed diagrams.

Throughout the book, each section is accompanied by a "Notes and References" section with further details and historical notes. At the end of the book, Appendix D presents a detailed subject classification of the theory of Korovkin-type theorems and a key that connects this classification to items in the bibliography. This bibliography is 50 pages long, contains about 700 entries, and appears to be comprehensive. An entry in the bibliography is often accompanied by its Mathematical Reviews number and its subject classification according to Appendix D. There is also a detailed index of symbols which I found useful in preparing this review.

It is clear from this impressive infrastructure that the authors have been meticulous in their efforts to produce this treatise on Korovkin-type approximation theory. The publishers have produced a book which is well bound, well printed, and pleasant to behold. This work by Altomare and Campiti is a comprehensive research monograph on Korovkin-type approximation theory. Since the authors go from the general to the particular, a graduate student starting out in the field may find it difficult to learn from this book in the initial stages of research. However, it would be a valuable reference to have available in the later stages of the project. University departments with active research workers in approximation theory should give serious consideration to including this book in the university's library.

TERRY M. MILLS

P. G. CIARLET AND J. L. LIONS (Eds.), *Handbook of Numerical Analysis*, Vol. III, North-Holland, Amsterdam, 1994, x + 778 pp., available in the USA/Canada from Elsevier, New York.

This book is the third in a sequence of volumes entitled *Handbook of Numerical Analysis*. I have not seen the first two volumes and I infer from the contents of this volume that there will be more in the sequence. In the general preface the editors write on p. v: "*The various volumes comprising the Handbook of Numerical Analysis will thoroughly cover all the major aspects of Numerical Analysis, by presenting accessible and in-depth surveys, which include the most recent trends.*" Perhaps the series would have been more appropriately named as "*Surveys in Numerical Analysis*" as the word "handbook" may suggest that this book is a compendium of algorithms or results.

Volume III contains five articles.

1. C. Brezinski, "*Historical perspective on interpolation, approximation and quadrature,*" pp. 3–46.
2. C. Brezinski and J. Van Iseghem, "*Padé approximations,*" pp. 47–222.
3. Bl. Sendov and A. Andreev, "*Approximation and interpolation theory,*" pp. 223–462.
4. P. Le Tallec, "*Numerical methods for nonlinear three-dimensional elasticity,*" pp. 465–622.
5. Bl. Sendov, A. Andreev, and N. Kjurkchiev, "*Numerical solution of polynomial equations,*" pp. 625–778.

Each article has its own subject index—a surprising but pleasing feature which was useful in preparing this review—and its own bibliography, which is extensive but not necessarily